

University of Saskatchewan

Department of Mathematics & Statistics

MATH 223.3 – FINAL EXAMINATION

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9 a.m.

CLOSED BOOK, NO CALCULATORS, NO ELECTRONIC DEVICES

The exam is in three parts, Part A, Part B and Part C.

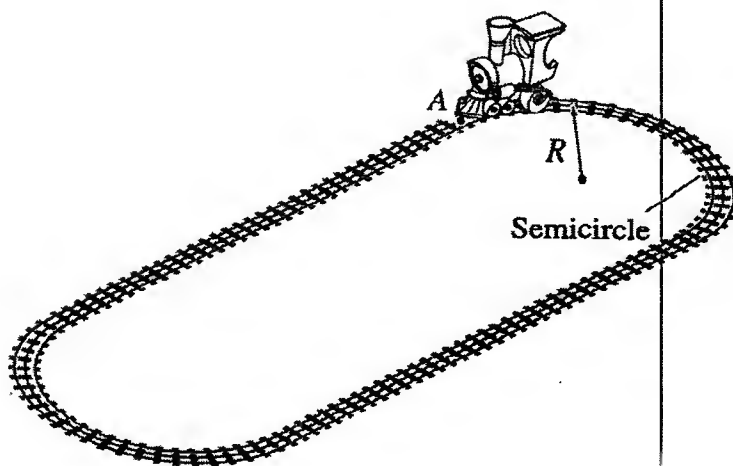
Use a different Exam Booklet for each part.

Please show all of your work. The marks for each question are shown by [x]

PART A. 34 MARKS.

Answer the following questions in an Exam Booklet and label it PART A.

- [7] 1. Find the area of the parallelogram with vertices $(1, 2, 3)$, $(4, 3, 7)$, $(-1, 3, 6)$, and $(2, 4, 10)$. Find the equation of the plane containing these points. *check vert*
- [4] 2. Find the distance between the line $x = 1 - t$, $y = 2 + 3t$, $z = 4 - 2t$ and the plane $2x + 4y + 5z = 10$. *check not parallel*
3. A particle has a trajectory defined by $x = t$, $y = t^2$, $z = t^2$, where t is time. Find its *velocity*, *speed* and *acceleration* at any time. What are the *normal* and *tangential* components of its *velocity* and *acceleration*?
- [6] 4. A toy train travels *at constant speed* around an oval track which consists of semi-circular segments joined together by straight segments as shown in the figure below.



This problem investigates why the train tends to wobble or even derail at the point A where the semi-circular segment joins the straight line segment of rail.

- The position of the train at time t can be described by a vector function \mathbf{r} . Find $\mathbf{r}(t)$ which describes the position of the train before the point A and after the point A.
- Show that, with your choice of $\mathbf{r}(t)$, the speed of the train is constant.
- Show that the acceleration is discontinuous at A.

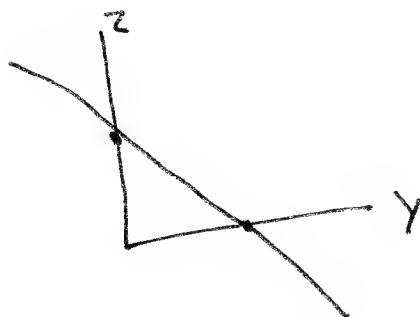
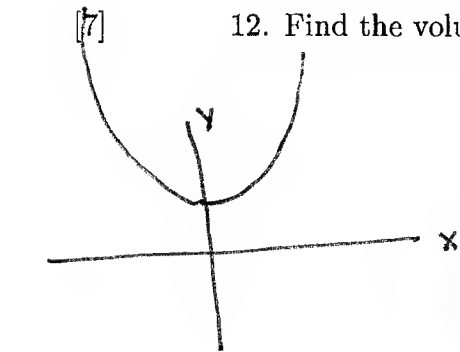
- [5] 5. Find the directional derivative of $f(x, y, z) = x^3 y \sin z$ at $(3, -1, -2)$ along the line $x = 3 + t, y = -1 + 4t, z = -2 + 2t$ in the direction of *decreasing* x .
- [5] 6. Find $\frac{dz}{du}$ if $z = x^2 y v^3$ and $x = u^3 + 2u, y = \ln(u^2 + 1)$, and $v = u e^u$.

PART B. 34 MARKS.

Answer the following questions in an Exam Booklet and label it PART B.

- [5] 7. Evaluate the double integral of $(4 - y^2)^{\frac{3}{2}}$ over the region bounded by the circle $x^2 + y^2 = 4$ in the first quadrant of the xy -coordinate plane.
- [5] 8. Use a double integral to find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2 - 2, y = 0$ about the line $y = -1$.
- [5] 9. Use double integrals to find the centroid of the region bounded by the curves $y = e^x, y = 0, x = 0$, and $x = 1$.
- [7] 10. Evaluate a double iterated integral to find the surface area of $x^2 + y^2 + z^2 = 2$ inside the cone $z = \sqrt{x^2 + y^2}$.
- [5] 11. Evaluate the triple integral of $x + y + z$ over the region V which is bounded by $z = 1 - x^2 - y^2$ and $z = 0$.
- Hint:
- (i) First show that the integrals of x and y are zero.
 - (ii) Integrate z by converting to a convenient coordinate system.

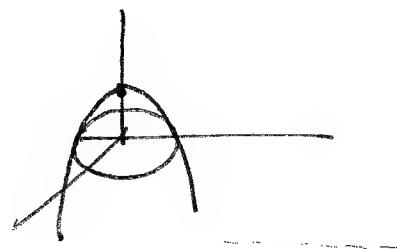
- [7] 12. Find the volume bounded by the surfaces $y + z = 2, z = 0$, and $y = x^2 + 1$.



$$\begin{aligned} z=0: & \quad x^2 + y^2 = 1 \\ z=1: & \quad x^2 + y^2 = 0 \\ z=2: & \quad x^2 + y^2 = -2 \\ z=-1: & \quad x^2 + y^2 = 2 \end{aligned}$$

$$z = -(x^2 + y^2 - 1)$$

$$\begin{array}{r} 15 \\ 3 \\ \hline 45 \\ -10 \\ +3 \\ \hline 35 \\ 3 \\ \hline 38 \\ 15 \end{array}$$



PART C. 32 MARKS.

Answer the following questions in an Exam Booklet and label it PART C.

- [8] 13. Verify the identities
- (a) $\text{curl}(\text{grad } f) = \nabla \times (\nabla f) = \mathbf{0}$
- (b) $\text{div}(\text{curl } \mathbf{F}) = \nabla \cdot (\nabla \times \mathbf{F}) = 0$
- where $f(x, y, z)$ is a sufficiently differentiable scalar field and $\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ is a sufficiently differentiable vector field
- [5] 14. Evaluate the line integral $\int_c ydx + xdy + zdz$, where c is the curve $z = x^2 + y^2$, $x + y = 1$ from $(1, 0, 1)$ to $(-1, 2, 5)$.
- [5] 15. Show that the line integral $\int_c 3x^2y^3dx + 3x^3y^2dy$ is independent of path. Evaluate it when c is the curve $y = e^x$ from $(0, 1)$ to $(1, e)$.
- [5] 16. Use Green's theorem to evaluate the line integral $\oint_c (xy^2 + 2x)dx + (x^2y + y + x^2)dy$, where c is the boundary of the region enclosed by $y^2 - x^2 = 4$, $x = 0$ and $x = 3$ and the direction of the circuit is counter clockwise.
- [9] 17. Find the point(s) on the surface $z^2 = 1 + xy$ closest to the origin $(0, 0, 0)$.
Hints:
- (a) Over what region in the xy -coordinate plane does the surface lie? In particular, what is the boundary of the region?
- (b) It is simpler to use $(\text{distance})^2$ since it is a minimum when (distance) is a minimum.

$$y^2 = \sqrt{4+x^2}$$

$$\begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x & z \end{vmatrix}$$

$$z_y = z_y - x_z \neq 0$$

END